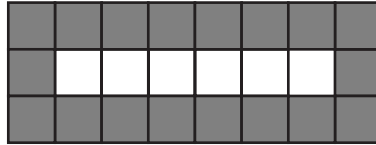A1 Lisa has some white square tiles and some grey square tiles.
They are all the same size.

She makes a row
of white tiles.

She surrounds the white
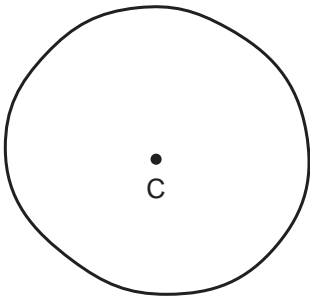tiles by a single layer
of grey tiles.

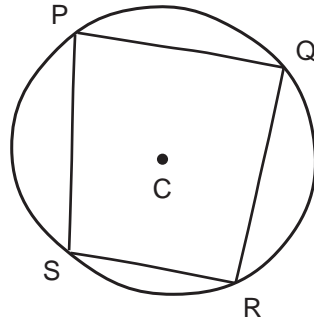How many grey tiles does she need to surround a row of 60 white tiles? . . . .

Show how you obtained your answer.

G1   Darren sketches a circle.   He then draws a quadrilateral   He then draws the diagonals
     He calls the centre C.   PQRS, whose corners lie on   of the quadrilateral.
                                  the circle.
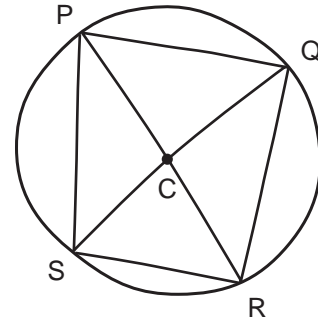
Darren says

"Whatever quadrilateral I draw with corners on a circle,
  the diagonals will always cross at the centre of the circle".

Is Darren right?   . . . .

Explain your answer.

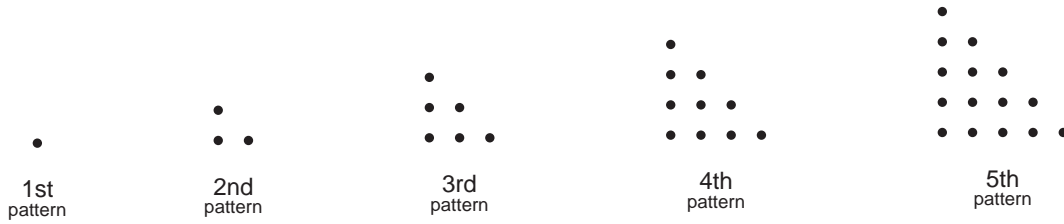A2   These are the first five patterns in a sequence of dotted triangle patterns:

1st
pattern

2nd
pattern

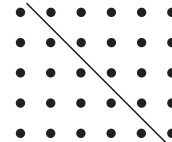3rd
pattern

4th
pattern

5th
pattern

a)   Rachel wants to *calculate* the number of dots in the 5th triangle pattern.

She draws the 5th pattern twice. She explains why:

"I can calculate the number of dots in this rectangle pattern.

I can then calculate the number of dots in the triangle pattern".

   i.   Show how Rachel calculates the number of dots in the rectangle pattern.

   ii.   Show how Rachel calculates the number of dots in the triangle pattern.

b)   Rachel wants to find the number of dots in the 20th triangle pattern.
She imagines drawing it twice to produce a rectangle pattern.

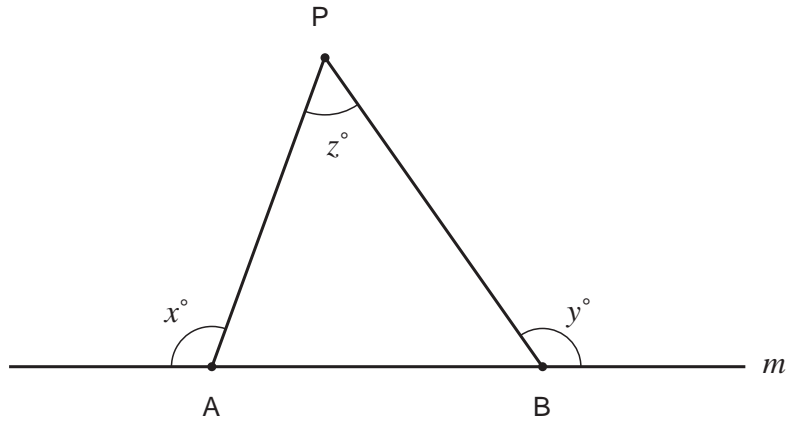Use the number of dots in the imagined rectangle pattern
to find the number of dots in the 20th triangle pattern.   . . . .
Show how you obtained your answer.

P

G3   In the diagram, A and B are two
fixed points on a straight line *m*.

Point P can move, but stays
connected to A and B
(the straight lines PA and
PB can stretch or shrink).

$z°$

$x°$ $y°$

A   B   *m*

Avril, Bruno, Chandra and Don are discussing whether this statement is true:

$x^{\bullet} + y^{\bullet}$ **is equal to** $180^{\bullet} + z^{\bullet}$.

Avril's answer

I measured the angles in the diagram and
found that angle *x* is 110˚, angle *y* is 125˚
and angle *z* is 55˚.

110˚ + 125˚ = 235˚,

and 180˚ + 55˚ = 235˚.

So Avril says it's true

Bruno's answer

I can move P so
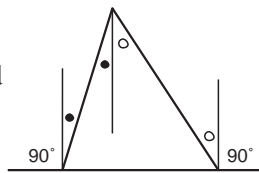that the triangle
is equilateral,
and its angles
are 60˚.

60˚

60˚   60˚
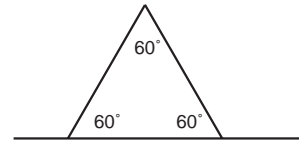
So *x* is 120˚ and *y* is 120˚.

120˚ + 120˚ is the same as 180˚ + 60˚.

So Bruno says it's true

Chandra's answer

I drew three parallel
lines.
The two angles marked
with a ● are the same
and the two marked
with a ○ are the same.

90˚   90˚

Angle *x* is 90˚+ ● and angle y is 90˚+ ○.

So *x* plus *y* is 180 + ● + ○, which is 180 + *z*.

So Chandra says it's true

Don's answer

I thought of a diagram where the angles *x*,
y and *z* are all 170˚.

170˚

170˚   170˚

So in my diagram *x* + *y* is not equal to 180 + *z*.

So Don says it's not true

---

a)   Whose answer is closest to what you would do?   . . . .

b)   Whose answer would get the best mark from your teacher?   . . . .

4   © 2000 Longitudinal Proof Project, Institute of Education, University of London

c)   For each of the following, circle whether you agree, don't know, or disagree.

The statement is:

$x^{\bullet} + y^{\bullet}$ **is equal to** $180^{\bullet} + z^{\bullet}$.

	agree	don't know	disagree
Avril's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Bruno's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Chandra's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Don's answer ...			
shows you that the statement is **not true**	1	2	3
shows you **why** the statement is not true	1	2	3

L1 Joe and Fred are thinking about the pair of numbers 3 and 11.

They notice that the SUM (3 + 11) is EVEN.

They notice that the PRODUCT (3 × 11) is ODD.

Joe says: If the SUM of two whole numbers is EVEN, their PRODUCT is ODD.

Fred says: If the PRODUCT of two whole numbers is ODD, their SUM is EVEN.

a) Are Joe's and Fred's statements saying the same thing?   . . . .

b) The PRODUCT of two whole numbers is 1271.

Suppose Fred is right.

Which one of these must also be right? Tick (✓) one box.
❏ You can be sure that the SUM of the two numbers is EVEN.
❏ You can be sure that the SUM of the two numbers is ODD.
❏ You can't be sure whether the SUM is ODD or EVEN until
    you know what the two numbers are.

c) Is Joe's statement true?   . . . .

Explain your answer.

d) Is Fred's statement true?   . . . .

Explain your answer.

A4   a)   4! means $4 \times 3 \times 2 \times 1$.

5! means $5 \times 4 \times 3 \times 2 \times 1$.

Is 5! exactly divisible by 3 ?   . . . .

Explain your answer.

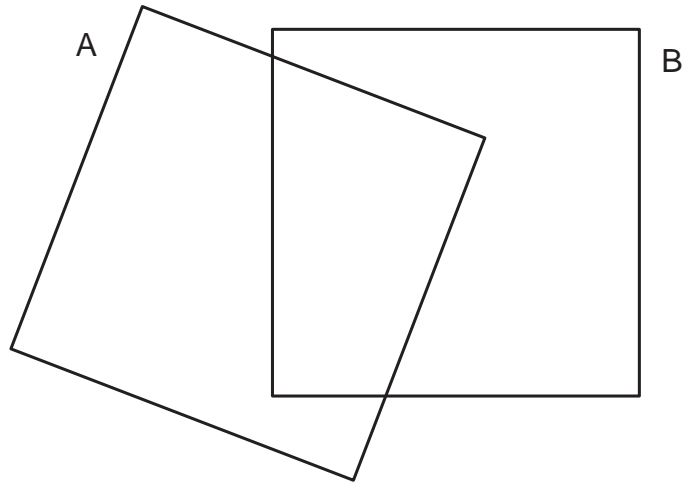b)   What does 100! mean?

c)   Is 100! exactly divisible by 31 ?   . . . .

Explain your answer.

G2   The diagram shows two
identical square tiles,
A and B.

A   B

The tiles overlap.

a)   Do the two non-overlapping
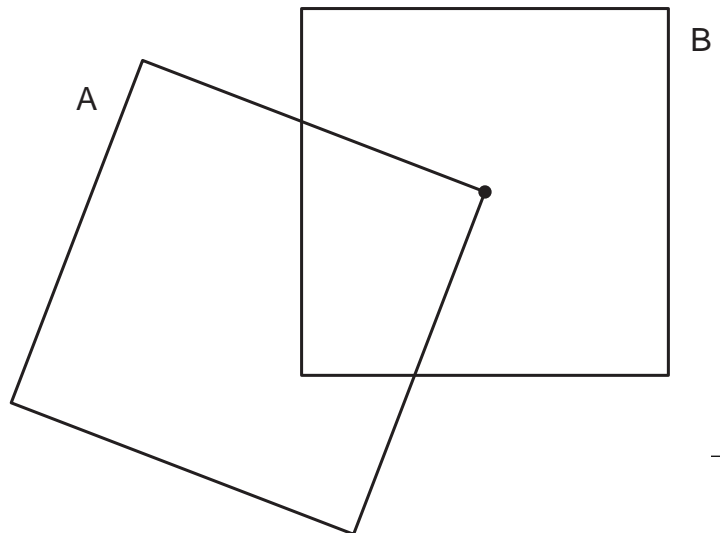regions have the same area?

. . . .

Explain your answer.

b)   One corner of tile A is moved
to the centre of tile B, as shown.
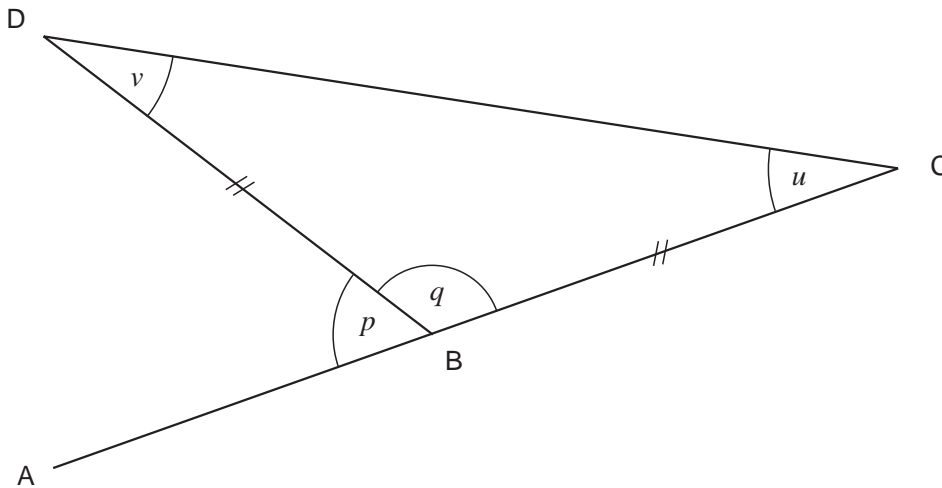
What fraction of tile B is
overlapped by tile A ?

. . . .

Explain your answer.

G4   In the diagram, ABC is a straight line and lines BD and BC are the same length.

a)   Find the size of angle u when angle p is 70˚.   . . . .

b)   Jenny wants to find the size of angle u when angle p is 76˚.
     J, K and L are her calculations, but they are not in the right order.

J   angle u + angle v = 180˚ − 104˚ = 76˚

K   angle u = 76˚ ÷ 2 = 38˚

L   angle q = 180˚ − 76˚ = 104˚

Write the letters J, K and L in the order in which Jenny did the calculations.   . . . . . .

c)   These are Jenny's reasons for her calculations, but they are not in the right order either.
     Match the reasons and calculations by writing the letters J, K, L in the blank circles:

◯   The base angles of an isosceles triangle are equal

◯   Angles on a straight line add up to 180˚

◯   The angle sum of a triangle is 180˚

A3   Five cards have the odd numbers 1, 3, 5, 7 and 9
      printed on one side, and the even numbers
      2, 4, 6, 8 and 10 printed on the other side.

1	3	5	7	9
2 on the other side	4 on the other side	6 on the other side	8 on the other side	10 on the other side
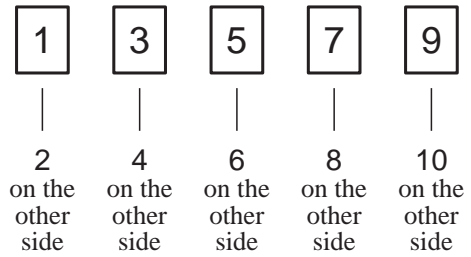
      The cards are dropped on the floor and spread out.

Amina, Ben, Carol and Davor are discussing whether this statement is true:

When two of the visible numbers are even, the five visible numbers add up to 27.

Amina's answer

I tried this example:

1, 4, 6, 7, 9.

I then tried two more examples. Each had two
even numbers and the total came to 27 each
time. I could try other examples with two even
numbers, they would come to 27 as well.

So Amina says it's true

Ben's answer

I tried all odd numbers first and got 25:

$1 + 3 + 5 + 7 + 9 = 25$.

If I change one odd number to an even number,
the total will be 1 bigger.
So if I have two even numbers, the total will
be 2 bigger.
So the total will be 27.

So Ben says it's true

Carol's answer

I wrote down these numbers:

1, 2, 3, 4, 9.

Two of the visible numbers are even but the
total is 19. So you do not always get 27.

So Carol says it's not true

Davor's answer

I thought of these as the visible numbers:

1, 3, 6, 8, 9.

Two of them are even and when I add all the
numbers I get 27.

So Davor says it's true

---

a)   Whose answer is closest to what you would do?   . . . .

b)   Whose answer would get the best mark from your teacher?   . . . .

A3   *Continued*

c)   For each of the following, circle whether you agree, don't know, or disagree.

The statement is:
When two of the visible numbers are even, the five visible numbers add up to 27.

	agree	don't know	disagree
Amina's answer …			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Ben's answer …			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Carol's answer …			
shows you that the statement is **not true**	1	2	3
shows you **why** the statement is not true	1	2	3

	agree	don't know	disagree
Davor's answer …			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

WAIT! *Please go back to any questions you left out, then check all your answers.*
After that, if there is any time left over, please answer this questionnaire:

Z1   a)   What did you feel about taking part in this survey?

      b)   Which question did you like best, and why?

      c)   Which question did you like least, and why?

      d)   Please add any other comments, if you wish to, about the survey.